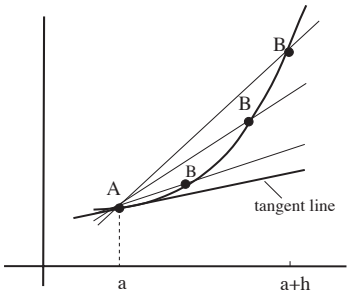
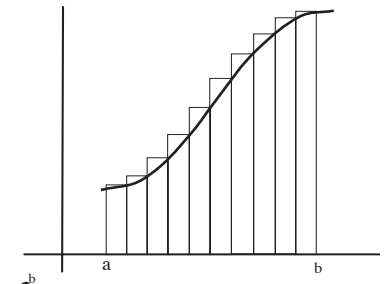
Sample Questions

PREPARING FOR THE

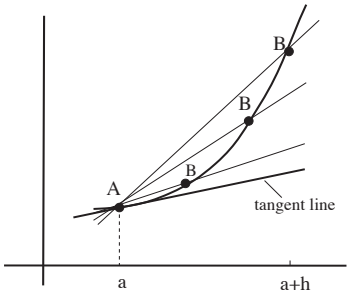
AP (AB) CALCULUS EXAMINATION



$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



$$\int_a^b f(x)\,dx = \lim_{n \to \infty} [f(x)\Delta x_1 + f(x)\Delta x_2 + f(x)\Delta x_3 + \ldots + f(x)\Delta x_n]$$



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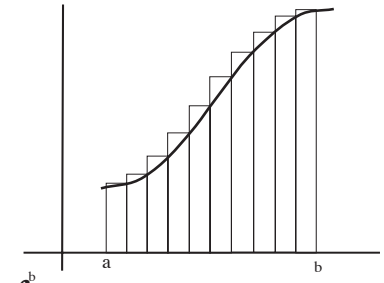

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Venture Publishing
9 Bartlet Street
Andover, MA 01810
Phone/Fax (508) 896-9486

Preface

This workbook is intended for students preparing to take the Advanced Placement Calculus AB Examination. It contains six practice tests that are based on the course description published by the College Board. We have tried to make each of the six tests in this workbook as much like the actual AP Exam as possible. For example, in the appropriate sections, there are questions that require students to make decisions about whether to use the graphing calculator a lot, a little, or not at all. In order to provide a greater supply of this type problem, our exams require the use of a calculator in about half the problems of Section I Part B, and all of Section II Part A.

Each student is expected to have a graphing calculator that has the capability to:

(1) produce the graph of a function within an arbitrary viewing window,
(2) find the zeros of a function,
(3) compute the derivative of a function numerically, and
(4) compute definite integrals.

In the free-response sections, solutions obtained using one of these four capabilities need only show the setup. Solutions using other calculator capabilities must show the mathematical steps that lead to the answer. In either case, a correct answer alone will not receive full credit.

As in the *AP Course Description for Mathematics*, our examinations are in two sections of equal weight. Section I is all multiple-choice and Section II is all free-response.

1. Section I Part A (30 questions in 60 minutes). Calculators may not be used in this part of the exam.
2. Section I Part B (15 questions in 45minutes). Calculators are allowed.
3. Section II Part A (2 questions in 30 minutes). Calculators are required
4. Section II Part B (4 questions in 60 minutes). Calculators may not be used and the student may go back to Part A if there is time.

In the hope of providing future students with a better workbook, the authors welcome your suggestions, corrections, problems of all sorts, and feedback in general. Please send your comments to:

Venture Publishing
9 Bartlet Street, Suite 55
Andover, Ma 01810
Phone/Fax 508-896-9486
E-Mail gwbest@tiac.net

EXAM I
CALCULUS AB
SECTION I PART A
Time–60 minutes
Number of questions–30

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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1. If $f'(x) = \ln(x - 2)$, then the graph of $y = f(x)$ is decreasing if and only if

   (A) $2 < x < 3$ (B) $0 < x$ (C) $0 < x < 1$ (D) $x > 1$

Ans ☐

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2. For $x \neq 0$, the slope of the tangent to $y = x \cos x$ equals zero whenever

   (A) $\tan x = -x$

   (B) $\tan x = \frac{1}{x}$

   (C) $\tan x = x$

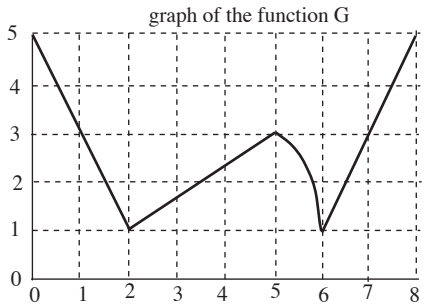
   (D) $\sin x = x$

Ans ☐

3. The function F is defined by

$F(x) = G[x + G(x)]$

where the graph of the function G is shown at the right.



The approximate value of $F'(1)$ is

(A) $\frac{2}{3}$

(B) -2

(C) -1

(D) $-\frac{2}{3}$

Ans ☐

4. $\int_2^6 \left(\frac{1}{x} + 2x\right)\, dx =$

(A) $\ln 4 + 32$

(B) $\ln 3 + 40$

(C) $\ln 3 + 32$

(D) $\ln 4 + 40$

Ans ☐

5. A relative maximum of the function $f(x) = \frac{(\ln x)^2}{x}$ occurs a

(A) 1

(B) 2

(C) e

(D) e^2

Ans ☐

6. Use a right-hand Riemann sum with 4 equal subdivisions to approximate the integral $\int_{-1}^{3} |2x - 3|\, dx$.

(A) 13

(B) 10

(C) 8.5

(D) 8

Ans ☐

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7. An equation of the line tangent to the graph of $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -3x + 1$

(B) $y = -3x - 7$

(C) $y = x + 5$

(D) $y = 3x + 1$

Ans ☐

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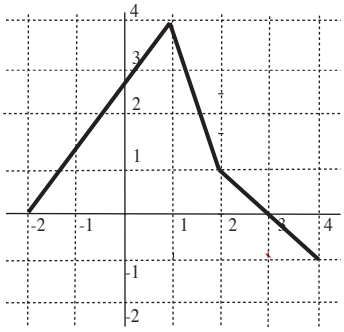
8 The graph of the function f, consisting of three line segments, is shown in the figure. Let

$$g(x) = \int_{-2}^{x} f(t)\, dt.$$

Then $g''(0) =$

graph of f



(A) $\frac{3}{2}$

(B) $\frac{4}{3}$

(C) 0 (D) –3

Ans ☐

EXAM I
CALCULUS AB
SECTION I PART B
Time–45 minutes
Number of questions–15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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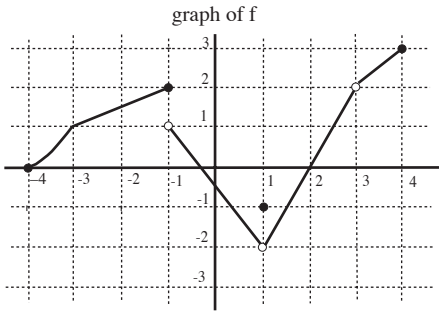
1. The function f is defined on the interval [–4, 4] and its graph is shown to the right. Which of the following statements are true?



I. $\lim_{x \to 1} f(x) = -1$

II. $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = 2$

III. $\lim_{x \to -1^+} f(x) = f(-3)$

(A) I only (B) II only (C) I and II only (D) II and III only

Ans

---

2. For $f(x) = \sin^2 x$ and $g(x) = 0.5x^2$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the instantaneous rate of change of f is greater than the instantaneous rate of change of g for which value of x?

(A) -0.8 (B) 0 (C) 0.9 (D) 1.2

Ans

3. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{1}{3}y^{-2}$ with initial condition $y(-1) = -1$?

(A) $y = (x-2)^{1/3},\ x > 0$

(B) $y = (x-2)^{1/3},\ x < 0$

(C) $y = x^{1/3},\ x < 0$

(D) $y = x^{1/3},\ x > 0$

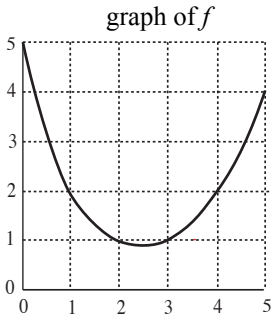
Ans

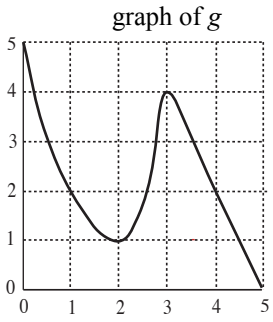
4. Find the average value of the acceleration of a particle whose velocity is modeled by the function $v(t) = t + 2\sin t$ on the interval [0, 2].

(A) 0.604 (B) 1.205 (C) 1.910 (D) 3.818

Ans

5. The graphs of functions f and g are shown at the right.





If $h(x) = g[f(x)]$, which of the following statements are true about the function h?

I. $h(0) = 4$.

II. h is increasing at $x = 2$.

III. The graph of h has a horizontal tangent at $x = 4$.

(A) I only (B) II only (C) I and II only (D) II and III only

Ans

CALCULUS AB
SECTION II, PART A
Time–30 minutes
Number of problems–2

A graphing calculator is required for some problems or parts of problems.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. Clearly label any functions, graphs, tables, or other objects you use. You will be graded on the correctness and completeness of your methods as well as your final answers. Answers without supporting work may not receive credit.

- Justifications require that you give mathematical (noncalculator) reasons.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

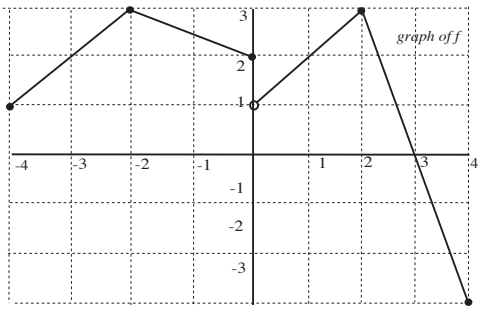
- If you use decimal approximations in your calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. Two functions, f and g, are defined on the closed interval $-4 \le x \le 4$. A graph of the function f is given in the following figure.



The table below contains some values of the continuous function g.

x	–4	–3	–2	–1	0	1	2	3	4
$g(x)$	10	9	5	–1	0	2	6	0	–3

(a) Find $f'(3)$.

(b) Approximate $g'(0)$. Show your work.

(c) If the function h is defined by $h(x) = g[f(x)]$, evaluate: i) $h(2)$ and ii) $h'(3)$

(d) Find $\int_0^4 f(x)\,dx$

---

2. A water tank holds 150 gallons of water at time $t = 0$. During the time interval $0 \le t \le 12$, water is pumped into the tank at the rate of

$$w(t) = 4e^{\sin(\frac{\pi}{12}t)} \text{ gallons per hour.}$$

At time $t = 8$ hours a second pump begins removing water from the tank at the rate of

$$R(t) = \frac{13t}{1+2t} \text{ gallons per hour.}$$

(a) How many gallons of water enter the tank during the time interval $0 \le t \le 8$?

(b) At what time during the time interval $0 \le t \le 8$ hours is the amount of water increasing most rapidly?

(c) What is the total amount of water in the tank at $t = 12$ hours?

(d) Is the amount of water in the tank increasing or decreasing at $t = 12$ hours? Justify your answer.

Time - 60 minutes
Number of problems - 4

A graphing calculator may NOT be used on this part of the examination.

- During the timed portion for part B, you may go back and continue to work on the problems in part A without the use of a calculator.

3. Car A has positive velocity $v(t)$ as it travels along a straight road, where v is a differentiable function of t. The velocity of the car is recorded for several selected values of t over the interval $0 \le t \le 60$ seconds, as shown in the table below.

t (seconds)	0	10	20	30	40	50	60
$v(t)$ (feet per second)	5	14	7	11	12	40	44

(a) Use the data from the table to approximate the acceleration of Car A at $t = 25$ seconds. Show the computation that lead to your answer. Indicate units of measure.

(b) Use the data from the table to approximate the distance traveled by Car A over the time interval $0 \le t \le 60$ seconds by using a midpoint Riemann sum with 3 subdivisions of equal length. Show the work that lead to your answer.

(c) Car B travels along the same road with an acceleration of $a(t) = \frac{1}{\sqrt{x+9}}$ ft / sec^2.

At time $t = 0$ seconds, the velocity of Car B is 3 ft/sec. Which car is traveling faster at $t = 40$ seconds? Show the work that lead to your answer

Answers

EXAM I SECTION I PART A

1.	A	11.	B	21.	C
2.	B	12.	D	22.	B
3.	D	13.	C	23.	D
4.	C	14.	B	24.	D
5.	D	15.	B	25.	B
6.	D	16.	E	26.	B
7.	A	17.	C	27.	C
8.	D	18.	C	28.	E
9.	B	19.	B		
10.	B	20.	A		

EXAM II SECTION I PART A

1.	C	11.	C	21.	D
2.	A	12.	B	22.	A
3.	B	13.	A	23.	C
4.	C	14.	D	24.	B
5.	D	15.	A	25.	B
6.	C	16.	C	26.	E
7.	E	17.	C	27.	A
8.	A	18.	E	28.	E
9.	B	19.	B		
10.	D	20.	D		

EXAM I SECTION I PART B

1.	D	7.	B	13.	C
2.	C	8.	B	14.	C
3.	C	9.	C	15.	B
4.	C	10.	C	16.	D
5.	D	11.	E	17.	E
6.	D	12.	E		

EXAM II SECTION I PART B

1.	E	7.	E	13.	D
2.	D	8.	A	14.	D
3.	D	9.	A	15.	B
4.	C	10.	A	16.	C
5.	C	11.	D	17.	D
6.	A	12.	C		

EXAM I SECTION II PART A

1. a) -3 b) $\frac{3}{2}$ c) i) 0 ii) $-\frac{9}{2}$ d) 4

2. a) $\int_0^8 4e^{\sin(\frac{\pi}{12}t)}\,dt = 68.236$ gallons

b) 6 hours

c) 220.116

d) Amount of water is decreasing since
W(12)– R(12) = –2.24 < 0

3. a) $0.4\,ft/\sec^2$ b) 1300 ft c) car A is faster

EXAM II SECTION II PART A

1. a) 1.764 b) 30.460 c) 3.671

2. a) 1004 gals

b) 39.541

c) 1073.99 gals